Foundations of Physics, Vol. 22, No. 12, 1992

Heisenberg Picture and Reality

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Received April 8, 1992

The idea is discussed according to which, in the Heisenberg picture, the operators correspond to the dynamic properties while the density matrix corresponds to our knowledge. A simple, soluble model is made use of in order to determine in what way this idea needs to be refined and what it then tells us about the relationship of reality and physics.

My thesis adviser was Louis de Broglie and I still keep many short letters he wrote me. They testify that, notwithstanding his many duties (and his innermost conviction that young people should find their own ways, as he himself did), he was most keen and gentle at guiding beginners. Of course I could hardly have found a thesis adviser with a more genuine interest in the foundations of quantum theory, and I am sure he would have inspired me with it if I had not felt it right from the start. It is a pleasure for me to be able to dedicate this article to his memory.

To the name "quantum theory" Louis de Broglie preferred that of "wave mechanics," which he had coined and which reflected his conception that the wave is quite a real thing. Of course, in this respect he and Schrödinger went hand in hand. The Schrödinger picture was for him the natural one as it is for many of us, and I doubt that the Heisenberg picture was ever, in his views, anything more than just an abstract tool without real physical content.

Still, in some fields this abstract tool has quite decisive advantages. It is difficult to imagine how quantum field theory could have been developed entirely without it. And even on questions having to do with the physical interpretation of the theory there are some arguments tending to show that

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0015-9018/92/1200-1495$06.50/0

it might help. After all, classical physics is interpretable without much problems and, in it, the dynamical variables are time-dependent. On this ground it would seem that a picture in which the mathematical objects describing these variables are time-dependent also (and obey classical equations) has some good chances of being more easily interpretable than one in which they are not. The fact that the alternative picture, the one of Schrödinger, suffers from well-known interpretational difficulties—entanglement and Schrödinger's cat—may be felt as adding weight to the foregoing idea.

The difficulties in question are intimately connected with measurement theory. An obvious question therefore is: "what about measurement—and knowledge—in the Heisenberg picture?" In the present paper I shall tackle this problem by means of a soluble model describing a measurement-like interaction. A convenient one has been considered by Peres.[1] It consists in a measurement of the z component S_z of a spin-1/2 particle S by means of an instrument A with pointer coordinate G. The eigenvalue equations

$$S_z|\pm\rangle = \pm|\pm\rangle \tag{1}$$

$$G|F_\pm\rangle = g_\pm|F_\pm\rangle \tag{2}$$

serve to define the notations. Let, moreover, the initial value, at time 0, of G be g_+, and let the measurement process consist in the fact that if $S_z = +1$ (in units of $\hbar/2$) the value of G is unchanged whereas if $S_z = -1$, G is flipped to $G = g_-$. An interaction Hamiltonian H' that, according to the time-dependent Schrödinger equation, accounts exactly for such a process is

$$H' = \beta(t)|-\rangle\langle-|\otimes P \tag{3}$$

where $\beta(t)$ is a smooth function of time with compact support $(0, t_1)$ satisfying

$$\int_0^{t_1} \beta(t)\, dt = \pi \tag{4}$$

and where P is the projector

$$P = 2^{-1}(|F_+\rangle - |F_-\rangle)(\langle F_+| - \langle F_-|) \tag{5}$$

for indeed the time-dependent ket

$$|\Psi(t)\rangle = \{\mathbb{1} + [e^{-i\alpha(t)} - 1]|-\rangle\langle-|\otimes P\}|\Psi(0)\rangle \tag{6}$$

with

$$\alpha(t) = \int_0^t \beta(t')\, dt' \tag{7}$$

is (as easily checked) a solution of the Schrödinger equation

$$i\frac{\partial|\Psi\rangle}{\partial t}=H'|\Psi\rangle \tag{8}$$

and, moreover, if

$$|\Psi(0)\rangle=|\Psi_+(0)\rangle\equiv|+\rangle\otimes|F_+\rangle \tag{9}$$

then Eq. (6) yields for the final state, $|\Psi(t_1)\rangle$, after the interaction is over:

$$|\Psi_+(t_1)\rangle=|+\rangle\otimes|F_+\rangle \tag{10}$$

while, if

$$|\Psi(0)\rangle=|\Psi_-(0)\rangle\equiv|-\rangle\otimes|F_+\rangle \tag{11}$$

then it yields

$$|\Psi_-(t_1)\rangle=|-\rangle\otimes|F_-\rangle \tag{12}$$

in agreement with the above stated conditions on G.

As shown by Eq. (6) [together with Eq. (4)], the unitary operator $U\equiv U(\infty)$ which, in the Schrödinger picture, describes the time evolution of the composite $S+A$ system from its initial to its final state is

$$\begin{aligned} U&=\mathbb{1}-|-\rangle\langle-|\otimes 2P\\ &=|+\rangle\langle+|\otimes(|F_+\rangle\langle F_+|+|F_-\rangle\langle F_-|)\\ &\quad+|-\rangle\langle-|\otimes(|F_-\rangle\langle F_+|+|F_+\rangle\langle F_-|) \end{aligned} \tag{13}$$

an operator which, in addition to being unitary, happens to be Hermitian as well: $U^\dagger=U$.

This model will now be "translated" into the Heisenberg picture. With the help of U it is an easy matter to study, in this picture, the time evolution of the operators representing the physical quantities that are of interest in the present problem. The most directly relevant ones are the operators $B(t)$ and $F(t)$ which, at time $t=0$ (i.e., before the measurement takes place) have the form

$$B(0)=(|+\rangle\langle+|-|-\rangle\langle-|)\otimes\mathbb{1} \tag{14}$$

and

$$F(0)=\mathbb{1}\otimes(g_+|F_+\rangle\langle F_+|+g_-|F_-\rangle\langle F_-|) \tag{15}$$

The operator $B(0)$ effectively operates in the Hilbert space of the measured spin-1/2 particle only, and adequately represents, at time $t=0$, the attribute "z spin component S_z" of the latter. Similarly $F(0)$ effectively operates in the Hilbert space of the instrument only and adequately represents, at time $t=0$, the attribute "pointer position G" of the latter. In Eq. (15) the eigenvalues g_+ and g_- of $F(0)$ are just the values the pointer coordinate can be found to have. Of course the actual values S_z and G are initially known to have are specified, not by the operators $B(0)$ and $F(0)$, but by the ket representing at time 0 the composite $S+A$ system. This ket is either $|\Psi_+(0)\rangle$ or $|\Psi_-(0)\rangle$ [Eqs. (9) and (11)] according to whether the to-be-measured spin points up or down. If we worked within the Schrödinger picture, this ket would be transformed by the S, A interaction constituting the measurement process into $|\Psi_+(t_1)\rangle$ [Eq. (10)] in the first case and $|\Psi_-(t_1)\rangle$ [Eq. (12)] in the second case. In the Heisenberg picture the representative ket remains equal to $|\Psi(0)\rangle$ since it is time-invariant during this whole interaction process and the operators change, according to

$$B(t_1)=UB(0)U=(|+\rangle\langle+|-|-\rangle\langle-|)\otimes\mathbb{1} \tag{16}$$

and

$$F(t_1)=g_+\hat{P}_+ + g_-\hat{P}_- \tag{17}$$

with

$$\hat{P}_+ = |+\rangle\langle+|\otimes|F_+\rangle\langle F_+| + |-\rangle\langle-|\otimes|F_-\rangle\langle F_-| \tag{18a}$$

and

$$\hat{P}_- = |+\rangle\langle+|\otimes|F_-\rangle\langle F_-| + |-\rangle\langle-|\otimes|F_+\rangle\langle F_+| \tag{18b}$$

as is easily calculated.

To calculate the probability p_i that the outcome of an observation of, say, G at time t_1 is g_i ($i=+$ or $-$) by using the Heisenberg picture, we use the spectral decomposition (17) of F. The probability p_i is then given by

$$p_i=\langle\Psi(0)|\hat{P}_i|\Psi(0)\rangle \equiv \mathrm{Tr}[|\Psi(0)\rangle\langle\Psi(0)|\hat{P}_i] \tag{19}$$

In the case in which $|\Psi(0)\rangle=|\Psi_-(0)\rangle$ (which is the only nontrivial one) formula (18b) then yields

$$p_+=0, \qquad p_-=1 \tag{20}$$

as expected, since the Schrödinger and Heisenberg pictures are mathematically equivalent. Nevertheless, Eqs. (17), (18a), and (18b) are instructive, for they show that $F(t_1)$ operates in the Hilbert space $\mathscr{H}^S \otimes \mathscr{H}^A$ of the whole $S+A$ system and contrary to $F(0)$ cannot be written in the form

$$\mathbb{1} \otimes K \tag{21}$$

where K would operate only in the Hilbert space $\mathscr{H}^A$ of the instrument. To describe this (well-known), general effect, the word entanglement is obviously appropriate, although the effect in question does not coincide with the one Schrödinger called entanglement since it takes place even in simple cases such as the above considered one, in which no initial linear superposition of states $|+\rangle$ and $|-\rangle$ is considered.

The above formulas may serve to investigate the bearing of an interpretation expressed in particular by Unruh[2] according to which the Heisenberg picture offers a decisive advantage over the Schrödinger picture when issues of quantum measurement theory are discussed. According to this interpretation, while the Schrödinger picture hopelessly mixes dynamics and "knowledge" (it is the same entity, ψ, that evolves according to the Schrödinger equation and is abruptly changed upon measurement), in the Heisenberg picture, on the contrary, one can regard quantum mechanics as actually giving attributes to the physical entities in the world, the only difference with classical physics being that these attributes are represented mathematically not by numbers but by operators. As is natural, these attributes change in the course of time and develop according to the appropriate laws of motion. Within this interpretation the wave function (or ket, or density matrix) essentially represents our knowledge of the system: not surprising that it should abruptly change when our information increases due to some measurement being done. But it is our knowledge that then abruptly changes, not the physical system.

It is clear that the explicit expressions (16) and (17) obtained in our model for the Heisenberg operators B and F are such as to raise some questions concerning the real import of the above interpretation. The point is that in a case in which we know for sure that at time $t=0$ the spin is down and G has value g_+, we also know for sure that at time t_1 the spin is down and G has value g_-. If we really were entitled to say, as we did above, that at time $t=0$ $B(0)$ [Eq. (14)] and $F(0)$ [Eq. (15)] are adequate representations of the *physical attributes of the* $S+A$ system, why should somebody coming in our laboratory at time t_1 and being *then* informed that the spin is down and G has value g_- not be entitled to say that it is at *that* time—not at time 0—that the right-hand sides of Eqs. (14) and (15) adequately describe the physical attributes in question? By describing our knowledge of the composite system by means of ket (12), this person would have a (Heisenberg) representation of both the system and our knowledge of it just as adequate at all times as the foregoing one, in which at time t_1 we described the composite system by means of expressions (16) and (17) and our knowledge of it by means of expression (11).

Since we have to do with the same system and the same "knowledge" in both cases, this plurality—or at least duality—of descriptions, which has no obvious parallel in the Schrödinger picture, is intriguing. At this stage, the only hope there is of salvaging the view that the operators represent the physical reality of the system and the ket represents our knowledge, seems to be to compare the state of affairs at hand to the fact that when we decide to represent a vector by its components we are free to choose the coordinate system in the way we like, and the sets of numbers we get depend on this free choice of ours. However, in the present case there is nothing that corresponds to the vector notation. The idea is sometimes entertained that the Heisenberg operators are just what would correspond to this coordinate-system-independent vector notation: but the above shows this to be a delusion.

With the foregoing in mind, we can now turn to measurement theory. Up to this point we have not been really dealing with it. It was mainly for semantic convenience that we referred to system A as an instrument. In fact, what we were interested in was just the behavior of a certain $S+A$ system (with a well-specified internal Hamiltonian) and the knowledge an external observer may have of it. At present let us go on considering the $S+A$ system as an ordinary quantum-mechanical one, but let us assume that its state at time 0 is a linear combination:

$$\Psi(0)=a\Psi_+(0)+b\Psi_-(0)=(a|+\rangle+b|-\rangle)\otimes F_+ \tag{22}$$

with

$$|a|^2+|b|^2=1 \tag{23}$$

This, of course, changes nothing to the Heisenberg equations of motion, so that at time t_1 Eqs. (16) and (17) are still valid (and remain so afterwards since the Hamiltonian then vanishes). Suppose now that at time t_1 we make (by means of some superinstrument) a measurement of G. The probability of getting g_- is again given by Eq. (19) with $i=-$ and is easily found to be $|b|^2$, as expected. Assuming that the outcome of the measurement is indeed g_-, the corresponding reduced density matrix ρ'_- is obtained from the initial one

$$\rho(0)=|\psi(0)\rangle\langle\psi(0)| \tag{24}$$

by the well-known formula

$$\rho'_-=\hat{P}_-\rho(0)\,\hat{P}_-/\mathrm{Tr}[\hat{P}_-\rho_0] \tag{25}$$

which yields

$$\rho'_{-} = |-\rangle\langle -| \otimes |F_{+}\rangle\langle F_{+}| \tag{26}$$

In Unruh's interpretation this ρ'_{-} is considered as expressing our knowledge of the $S+A$ system after t_1. To be sure, this knowledge is expressed there in a queer and roundabout way since there is an $|F+\rangle\langle F+|$ symbol in Eq. (26) while what we know is that $G=g_{-}$. But, again, this is because Eq. (26) must be used in conjunction with the description of the $S+A$ system given by Eqs. (16) and (17) so that, for us who know that $G=g_{-}$ at t_1, the probability to get $G=g_{-}$ again upon measuring G once more at a time $t_2 > t_1$ is

$$\mathrm{Tr}[\rho'_{-}\hat{P}_{-}] = 1$$

as it should.

Unruh appropriately stresses the fact that in the Heisenberg picture the transition from ρ to ρ' has nothing to do with the dynamics of the system, which is expressed by the time dependence of the Heisenberg operators. This he contrasts with the elementary interpretation of the collapse in the Schrödinger picture, where the wave function is thought of as representing reality and where its collapse is therefore conceived of as an event that affects the system itself. But the question whether the advantage of the Heisenberg picture is as great as he sees it to be is a subtle one and is worth further examination. For indeed, also in the Schrödinger picture nothing forces us to consider the collapse as a real physical effect. In fact, we even know that as long as the instrument (here the superinstrument) is a system with a finite number of degrees of freedom and can be thought of as isolated, we are not allowed to do so. In other words, in the Schrödinger picture and when questions of interpretation are at stake, we should consider not just one but two wave functions: the reduced one that encodes what we consider as being our collective knowledge, just as ρ'_{-} does here, and the unreduced, global one that represents the dynamics of the system *plus* instrument. That the same duality is present also in the Heisenberg picture is clearly seen on the above model for along with ρ'_{-}, as generally expressed by Eq. (25) [and in the present instance by Eq. (26)], we may also consider the—more natural—

$$\rho''_{-}(t_1) = |-\rangle\langle -| \otimes |F_{-}\rangle\langle F_{-}|$$

which has to be used together with the "collapsed" dynamics

$$B(t \geqslant t_1) = (|+\rangle\langle +| - |-\rangle\langle -|) \otimes \mathbb{1}$$

$$F(t \geqslant t_1) = \mathbb{1} \otimes |F_{-}\rangle\langle F_{-}|$$

It is true that contrary to the Schrödinger global wave function, the Heisenberg picture operators can be written down without any symbol referring to the variables of the instrument with the help of which the information described by Eq. (25) is gained. But this is essentially due to the fact, to which we return somewhat more explicitly below, that contrary to what is sometimes believed, these operators do not really correspond to any actual situation—or state—of the system, whereas the global Schrödinger wave function is, rightly or wrongly, sometimes understood that way (see, e.g., Everett's theory).

These considerations may be thought to lessen the advantage that the Heisenberg picture seemed to have over the Schrödinger one concerning the problems of interpretation. But this appreciation requires qualifying again. What seems really doubtful is the possibility of bluntly asserting that in the Heisenberg picture the operators correspond to the physical atributes of the systems roughly in the same way as appropriate numbers correspond to them within classical physics. The point here is that in classical physics symbols such as **r**, **v**, **E**, **B** etc. have a dual role. They serve for writing down the dynamical equations (Maxwell's equations and the rest) *and* for designating the values that, in any particular instance, the quantities they refer to actually have (of course these values differ from one instance to the next: Not all planetary systems have their planets exactly at the same points at the same time, and so on). If the symbols $B(t)$, $\hat{P}(t)$, etc. that appear in the Heisenberg picture were to play in quantum physics the role these classical symbols play in classical physics, they should somehow have the same dual use: It would not be enough that they serve in writing down general equations derived from some Lagrangian. They should also, in such and such particular instance, serve to designate the (operator-valued) physical quantity that, in this instance, the considered system *actually has*. Indeed, when, in classical physics, we say that a measurement (assumed ideal) "merely modifies our information *without changing the values* of **r**, **v** and the other *dynamical variables of the system*," we obviously refer to this second role the symbols **r**, **v**, etc. have. For us to be able to give a meaning to the assertion that a quantum measurement does not change the dynamical quantities represented by the Heisenberg operators $B(t)$, $\hat{P}(t)$, etc. (and only changes our knowledge concerning them), it would be necessary that the same should be true of them. In other words, these operators should be actually interpretable as representing some properties that in such and such circumstances a system happens to have. But this means that somehow they should refer, at least in a sense, to the physical state of the system as it, contingently, is. The illusion that they do seems to be implicitly entertained by some authors. It presumably stems from the fact that in a case in which two systems, such as S and A in our model, come into interaction, it seems extremely natural to describe the overall system they constitute before they interact by means of a Cartesian product of two operators, each one of which operates in the Hilbert space of one system only. A partial specification of the involved operators is thereby produced that goes over and beyond the mere general formulas derived from the Lagrangian of the theory, and takes into account at least some of the contingent, physical properties of the systems at hand. But, as our simple model clearly shows, this specification is in fact just a convenient choice *we* make. In the model in question it can be made either at time $t=0$ or at time $t=t_1$, but not at both, and there is no reason whatsoever to consider that the contingent physical features (or "dynamical properties") of the systems at hand make one of these choices more natural than the other.

Finally, therefore, we come to the conclusion that the Heisenberg operators reflect *no* contingent features, or dynamical properties, of this or that particular system, which means that the above-considered analogy with the coordinates of a vector was still overemphasizing the similarity between these operators and classical dynamical variables. But—and this is, I think, the most important point—all this does not mean that these operators have no correspondence with reality. Their role and usefulness in the *general equations* of the theory amply testify to the contrary. Only, this is not the contingent reality of this or that particular physical system. It is the structural reality—so to speak—of the world. Hence, the existence and usefulness of the Heisenberg picture is a good indication of the fact that the one word "reality" covers in fact two different concepts both of which are meaningful: there is, on the one hand, the just-mentioned structural reality of the world, which I use to call "independent reality" (but I shall not quarrel about this name any more than about any other one!), and, on the other hand, the observable, contingent reality of this or that particular system: the one that our wave functions, density matrix, and so on tell us something about. That this "empirical reality," as I call it, is intersubjective rather than strongly objective is, I think, amply demonstrated by the detailed investigations that have been carried out by many people in the foundations of quantum mechanics (see, e.g., Ref. 3).

In 1914 Henri Poincaré, in *La Science et l'Hypothèse*, spoke of the "real objects" as objects that "Nature will hide from us forever." But, he went on, this does not mean that the status of the physical theories is thereby lowered to that of mere recipes, for, he wrote, the equations remain true even when their interpretation change and they express *relationships* between the objects. If they remain true it is because these relationships are real. Seen in the light of this philosophy, the operators in the Heisenberg picture appear as elements of a true description of the structure—*not* produced by man—of "independent reality." And the fact that the Heisenberg picture works therefore indicates that, contrary to the views of "antirealists" and other philosophers with similar trends, the notion of an independent reality makes sense, even though no kind of "naïve realism" seems able to survive the findings of contemporary physics.

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